Tayebi

Engineering 0138: Advanced control II

Mock Exam

Thursday, April 25th, 2002
9:00AM To 12:00N
Closed book
Do all your work on sheets provided.

Q-1 [7]

Consider the following transfer function system

$$\frac{Y(s)}{U(s)} = \frac{s+1}{s^2+10s+25}$$

Obtain the state space representation of this system in

a) Controllable canonical form.
b) Observable canonical form
c) Diagonal or Jordan form

Q-2 [5]

Consider the system defined by

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

Where

$$A = \begin{bmatrix} 1 & 3 \\ -1 & -2 \end{bmatrix},\ B = \begin{bmatrix} 4 \\ 5 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Transform the system into the controllable canonical form.

Q-3 [18]

Consider the system defined by

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

Where

$$A = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

a) Check the controllability and observability of the system.
b) Design a state feedback tracking controller of the form

$$u = -Kx + K_I \int_0^t (y_d(\tau) - y(\tau))d\tau,$$

in order to have the closed-loop poles at $(-4 \pm 4j)$, (-1) and the steady state error equal to zero for a step input.

c) Design a state observer for the system with the following performance:

-Settling time at 2%=1s

-Damping ratio=0.7.

Q-4 [12]

Consider the system defined by

$$\dot{x} = Ax + Bu \qquad x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$y = Cx$$

Where

$$A = \begin{bmatrix} 1 & 3 \\ -4 & -6 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

a) Design a state feedback stabilizing controller using LQR with

$$R = 1 \text{ and } Q = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix};$$

Note that the matrix P solution of the Riccati equation is given by

$$P = \begin{bmatrix} 15.4929 & P_{12} \\ -2.7205 & P_{22} \end{bmatrix}$$

Find P_{12} and P_{22}, and give the expression of your state feedback.

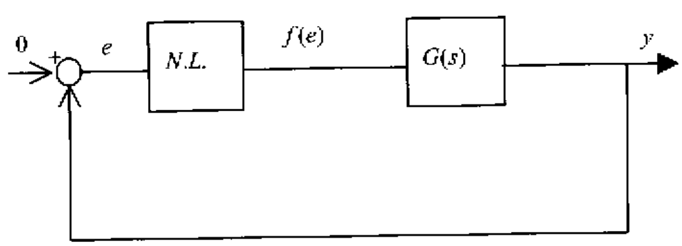
b) Give the minimum value of the cost function

c) Give the resulting closed loop system.

d) Solve the closed-loop system equations to determine the explicit expression of $y(t)$

Q-5 [8]

Consider the following closed loop system



With $G(s) = \dfrac{K_1}{s(T_1 s + 1)(T_2 s + 1)}$.

The nonlinear element is a relay with the following describing function

$$N(A) = \frac{-4h}{\pi A}$$

a) Discuss the existence of limit cycles according to the parameters K_1, T_1 and T_2.

P.S.

Given Laplace transorms:

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \rightarrow \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin\left(\omega_n \left(\sqrt{1-\xi^2}\right) t\right)$$

$$\frac{s}{s^2 + 2\xi\omega_n s + \omega_n^2} \rightarrow \frac{-1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin\left(\omega_n \left(\sqrt{1-\xi^2}\right) t - \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right)$$